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Effects of disorder on collective modes in single- and double-layer Bose systems

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Abstract. We report some new results from our study of two-dimensional (2D) Bose systems, particularly the collective excitations in 2D charged Bose gas, in the presence of disorder. The effects of disorder are taken into account through collisions the Bose particles suffer due to disorder; these collisions are considered in the relaxation-time approximation within a number-conserving scheme in the random-phase approximation (RPA) at $T = 0$. The dependence of the plasmon dispersion on the boson interparticle potential, and the critical wave vector below which the plasma excitations cannot propagate, are investigated. The static structure factor $S(q)$ in the presence of disorder is evaluated in closed form. We consider single- and double-layer systems and compare our results with the corresponding electron gas systems. A double-layer system in which one layer is disordered while the other is disorder-free exhibits results analogous to the 'drag effect'. Many-body extensions beyond the RPA are also discussed.

1. Introduction

The interest in two-dimensional electronic systems such as electrons at the interfaces of semiconductors and similar devices has a long and rich history [1]. Studies on low-dimensional Bose systems, on the other hand, have not received as much attention in recent years as low-dimensional Fermi—especially electron and hole—systems. However, Bose systems are interesting in their own right, and in the context of a possible theory of high-temperature superconductivity low-dimensional Bose systems have been invoked [2–5]. An experimental realization of a low-dimensional charged Bose gas in condensed-matter systems such as layered superconductors is thought to be possible through the formation of bipolarons. A reasonably strong coupling of electrons with phonons (or with some other neutral bosonic excitations) leads to polarons. The latter can attract each other via an induced lattice distortion if the resultant attractive interaction overcomes the Coulomb repulsion, and can form bipolarons [2, 5]. We may regard the bipolaronic carriers as charged Bose particles. Dielectric properties of charged bosons were recently studied by Conti *et al* [6]. In a previous paper [7] we have discussed two important many-body properties, namely collective modes (plasmons) and screened interactions in a pure charged Bose gas (CBG) system which is free of disorder or impurity.

It is known [8–10] that disorder, taken into account in the form of carrier–impurity collisions, severely constrains plasmon propagation in a single-layer electron gas, and that this constraint may be partially remedied in a double-layer electron gas. The main finding is that in lower dimensions (lower than three) the plasmon dispersion in the absence of

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disorder becomes gapless and develops a cut-off in wave vector in the presence of disorder; collective excitations having wave vectors less than the cut-off value cannot propagate.

In this paper we study the effects of disorder on collective modes in two-dimensional (2D) Bose systems and present new results. One distinguishing feature of the Bose systems that we consider is that at $T = 0$ all of the particles are assumed to be in a condensate state both in the absence and in the presence of disorder. We neglect the depletion of the condensate due to interactions and disorder. It was argued that disorder cannot completely deplete the condensate [11]. In our study we do not consider a strong disorder: the latter is expected to lead to a localization of bosons [11]. For clean (disorder-free) two-dimensional CBG systems we have reported [7] that the plasmon characteristics are similar to those found for the corresponding electron gas systems. We believe that it would be of interest to perform a similar study in the presence of disorder. Dielectric properties of a disordered Bose condensate were also investigated within the memory-function formalism [12]. Our approach is phenomenological in the sense that disorder is treated within the relaxation-time approximation, with a parameter τ appearing in the density response function. For a constant τ it gives the same result as the memory-function formalism. Although we do not specify the origin of disorder, it may arise from scattering mechanisms such as impurity scattering, and carrier–carrier or carrier–phonon interactions. More realistically, a momentum-independent [2] τ may be regarded as describing collisions with point defects (or acoustic phonons).

The rest of this paper is organized as follows. In the next section we introduce the effects of disorder within a number-conserving scheme to calculate the plasmon dispersion of a 2D-CBG. We also investigate the role of the interaction potential in the properties of plasmons. An analytic expression is given for the static structure factor $S(q)$ for a disordered CBG. Plasmons in a double-layer, disordered CBG are studied in section 3. Intralayer and interlayer static structure factors are calculated. To compare our results with the CBG, we also calculate the plasmon dispersions in a double-layer, 2D electron gas in section 4. Finally, in section 5 we conclude with a discussion of our results.

2. Collective modes of a 2D disordered Bose system

2.1. Plasmon dispersion

We consider a single-layer, one-component, disordered Bose gas. At zero temperature, the system is assumed to be in the condensate phase both in the absence and in the presence of disorder. The density–density response function for an interacting system of bosons within the random-phase approximation (RPA) is given by $\chi(q,\omega) = \chi^0(q,\omega)/[1 - v_q\,\chi^0(q,\omega)]$, in which the response function for a non-interacting system without disorder at $T = 0$ is

$$\chi^0(q,\omega) = \frac{2n\epsilon_q}{(\omega + \mathrm{i}\eta)^2 - \epsilon_q^2} \tag{1}$$

with the free-particle energy $\epsilon_q = q^2/2m$, and η a positive infinitesimal quantity (we take $\hbar = 1$). v_q is any physically reasonable (Fourier-transformable) potential for a 2D system. It may be recalled that the plasmon dispersion for a 2D-CBG interacting via a $(1/r)$-potential, obtained from the poles of the RPA density–density response function, yields the Bogoliubov result

$$\omega_{\mathrm{pl}}(q) = E_s[x + x^4]^{1/2} \tag{2}$$

with $x = q/q_s$, and $E_s = q_s^2/2m$. Here we have defined the screening wave vector of the Bose condensate $q_s = (8\pi n/a_B)^{1/3}$. Defining a dimensionless density parameter

$r_s^2 = 1/(\pi n a_B^2)$, where a_B is the effective Bohr radius, and n is the 2D density of bosons, we can express the screening wave vector as $q_s a_B = 2/r_s^{2/3}$.

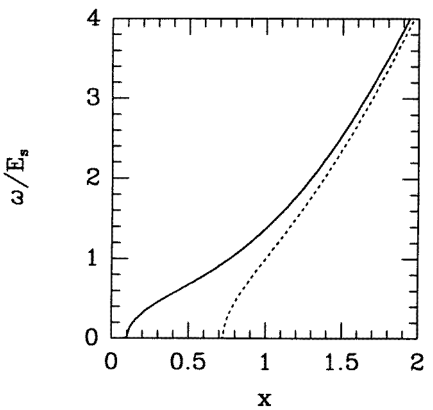

Figure 1. Plasmon dispersions of a disordered single-layer CBG for $1/(2\tau E_s)^2 = 0.1$ (solid) and $1/(2\tau E_s)^2 = 1$ (dotted).

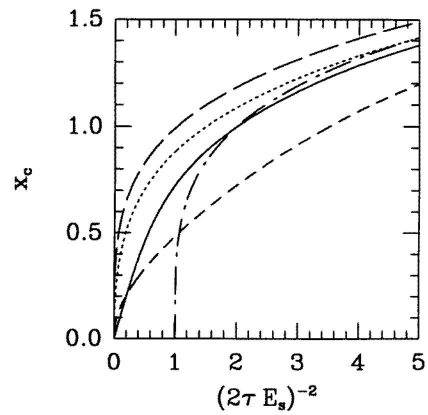

Figure 2. The critical wave vector q_c above which plasmons can propagate as a function of $1/(2\tau E_s)^2$. The parameters used for each curve are described in the text.

The effect of disorder in the many-body dynamics of the Bose system will be included in the form of collisions the Bose particles may undergo in the presence of disorder. The collisions will be treated in the relaxation-time approximation. The RPA polarizability in this approximation and in the number-conserving scheme is given by [13–15]

$$\chi_0^*(q,\omega;1/\tau) = \frac{\omega_+\,\chi_0(q,\omega_+)}{\omega + (\mathrm{i}/\tau)\chi_0(q,\omega_+)/\chi_0(q,0)} = \frac{2n\epsilon_q}{\omega\omega_+ - \epsilon_q^2} \tag{3}$$

where $\omega_+ = \omega + \mathrm{i}/\tau$ (τ is the relaxation time which will be treated as a phenomenological parameter), and $\chi_0(q,0)$ is the static susceptibility. In the limit $\tau \to \infty$, $\chi_0^*(q,\omega;1/\tau \to 0)$ becomes the collisionless $\chi_0(q,\omega)$. The plasmon dispersion for a 2D disordered Bose condensate takes the form

$$\omega_{\mathrm{pl}}(q) = \left[\epsilon_q^2 + 2n\epsilon_q v_q - \frac{1}{4\tau^2}\right]^{1/2} - \frac{\mathrm{i}}{2\tau} \tag{4}$$

for any v_q as specified earlier. We note that the above expression is valid within the RPA for all wave vectors, since no approximations were made to the density response function. Disorder effects (i.e., finite τ) tend to reduce the plasmon dispersion. Similar conclusions were drawn by Gold [16] within the memory-function approach to treating the disordered Bose condensate. Taking $v_q = 2\pi e^2/q$, we study the dependence of plasmon dispersion on the relaxation time for a 2D-CBG as shown in figure 1. The imaginary part to the plasmon dispersion is independent of q within the RPA, a situation different from that for the electron gas case in which it is wave-vector dependent [8].

As in the case of 2D electron gas [8, 9], there exists a critical wave vector q_c below which the collective modes do not propagate. The critical q_c is obtained from the solution of

$$q_c^4 + 4nmq_c^2 v_{q_c} - \left(\frac{m}{\tau}\right)^2 = 0. \tag{5}$$

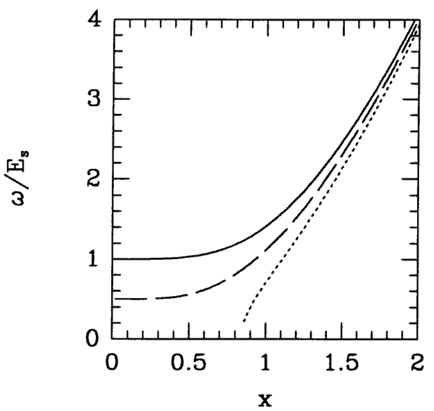

Figure 3. Plasmon dispersions for a logarithmic potential for $1/(2\tau E_s) = 0$, 0.75, and 1.5, indicated by solid, dashed, and dotted lines, respectively.

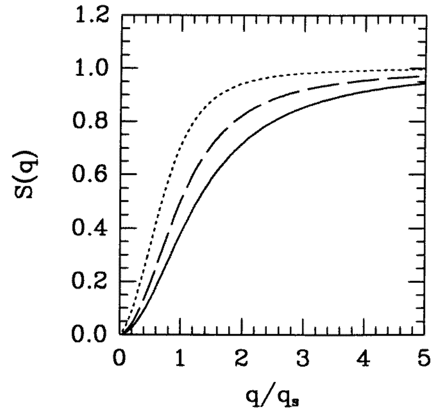

Figure 4. The static structure factor $S(q)$ for a single-layer CBG within the RPA and $(2\tilde{\tau})^{-2} = 0$, 1, and 5 shown by dotted, dashed, and solid lines, respectively.

To get more insight into the dependence of q_c on the interaction potential and the relaxation time τ we investigate several cases. For the Coulomb $(1/r)$-potential in 2D, the Fourier component is $v_q = 2\pi e^2/q$, and q_c is obtained from the above fourth-degree equation, i.e., $x_c^4 + x_c - 1/(4\tilde{\tau}^2) = 0$, where we have used $x_c = q_c/q_s$, and $\tilde{\tau} = \tau E_s$. If a short-ranged potential (a δ-function in real space, and constant V_0 in momentum space) described by the parameter $\gamma = 2nV_0/E_s$ is considered, we obtain

$$x_c = \left[-\frac{\gamma}{2} + \frac{1}{2}\left[\gamma^2 + \frac{1}{\tilde{\tau}^2}\right]^{1/2}\right]^{1/2}. \tag{6}$$

A dipole interaction of the form $v(r) = (p^2/r^3)[1 - \exp(-r^2/d^2)]$ where p is the dipole strength, was recently studied by Kachintsev and Ulloa [17] to describe 2D excitons. For comparison, we also employ this interaction, of which the Fourier transform v_q is well behaved [17]. On the other hand, a logarithmic potential [18] $v(r) = -e^2 q_s \ln r$, with Fourier transform $v_q = 2\pi e^2 q_s/q^2$, yields

$$x_c = \left(\frac{1}{4\tilde{\tau}^2} - 1\right)^{1/4} \tag{7}$$

for the cut-off wave vector. In figure 2 we show x_c for various models of the interaction potential v_q in a 2D Bose system as a function of $(4\tilde{\tau}^2)^{-1}$. Solid, dotted, dashed, and chain lines indicate Coulomb $(1/r)$-, short-ranged, dipole, and logarithmic interactions, respectively. We observe that as $\tau \to \infty$, all curves except that for the logarithmic interaction approach $q_c = 0$, indicating that plasmon dispersion starts from $q = 0$ in this limit. The logarithmic potential is somewhat different to the others in the sense that even in the absence of collisions the plasmon dispersion for long wavelengths has a gap (as in the 3D case), i.e., $\omega_{\mathrm{pl}}/E_s = (1 + x^4)^{1/2}$. This is in agreement with previous findings for this potential [18, 19]. The chain curve in figure 2 implies a softening of the logarithmic potential plasmons beyond a critical wave vector q_c only when $(4\tilde{\tau}^2)^{-1} > 1$. To see how the relaxation time affects the plasmon dispersion for a logarithmic potential, we show in figure 3 the collective mode as a function of x for various values of $(2\tilde{\tau})^{-2}$.

We finally consider an interaction potential of the form $v(r) = V_0 K_0(r/\lambda)$, where V_0 is a constant, $K_0(x)$ is the zeroth-order modified Bessel function, and λ a range parameter.

Such an interaction has been used in connection with the dynamics of flux-line lattices in superconducting systems [20]. For short distances ($r \to 0$) the above interaction behaves as $v(r) \sim \ln r$. The Fourier component is given by $v_q = 2\pi V_0/(q^2 + \lambda^{-2})$. Note that for $\lambda \to \infty$, we get $v_q \sim 1/q^2$, 3D-like behaviour. If we now look at the critical wave vector q_c above which plasmons propagate, we need to solve

$$x_c^4 + \Lambda \frac{x_c^2}{x_c^2 + \tilde{\lambda}^{-2}} - \frac{1}{4\tilde{\tau}^2} = 0 \tag{8}$$

where we have defined $\Lambda = 4nV_0/(E_s q_s^2)$, and $\tilde{\lambda} = \lambda q_s$. The short-dashed line in figure 2 shows x_c as a function of the relaxation-time parameter for $\tilde{\lambda} = 1$ and $\Lambda = 5$. We observe that the qualitative behaviour of x_c for the $K_0(x)$-potential is similar to that in the previous examples except for for the $(\ln r)$-potential. We can perhaps understand this qualitative difference by noting that $v(r)$ behaves as $\ln r$ only at short distances, whereas the collective excitations are more concerned with long distances (i.e., small-q behaviour).

2.2. Static structure factor

The static structure factor $S(q)$ for a single-layer CBG may be calculated by using the relation [21]

$$S(q) = -\frac{1}{n\pi} \int_0^\infty \mathrm{d}\omega \; \chi(q, \mathrm{i}\omega) \tag{9}$$

where the density–density response function is of the form $\chi = \chi_0^*/[1 - v_q \chi_0^*]$ in the RPA, where χ_0^* which includes the effects of disorder has already been introduced in equation (3). We calculate $S(q)$ for a 2D-CBG for which $v_q = 2\pi e^2/q$, and obtain [22] $S(q) = (1/\pi)\epsilon_q I(\Delta)$, where

$$I(\Delta) = 2 \begin{cases} \dfrac{1}{\sqrt{\Delta}} \left[\dfrac{\pi}{2} - \tan^{-1}\left(\dfrac{1/\tau}{\sqrt{\Delta}} \right) \right] & \text{for } \Delta > 0 \\ \tau & \text{for } \Delta = 0 \\ \dfrac{1}{\sqrt{-\Delta}} \tanh^{-1}\left(\dfrac{1/\tau}{\sqrt{-\Delta}} \right) & \text{for } \Delta < 0 \end{cases} \tag{10}$$

and $\Delta = 4(\epsilon_q^2 + 2nv_q\epsilon_q) - 1/\tau^2$. $S(q)$ for the disorder-free limit, i.e., when $1/\tau \to 0$, can be obtained either from the first expression in equation (10) or from equation (9) directly, and we find $S(q) = [1 + 2nv_q/\epsilon_q]^{-1/2}$ in agreement with the previously known result [23]. The effect of finite τ, i.e., disorder on $S(q)$, is illustrated in figure 4. As $1/\tau$ gets larger with increasing disorder, the magnitude of $S(q)$ becomes smaller, and it approaches unity in the large-q limit in a slower fashion than for a disorder-free system. For the disordered system, another interesting feature of $S(q)$ is that it is a smooth function of the disorder parameter and does not reflect the presence of a sharp cut-off q_c in the plasmon dispersion. The dotted line in figure 4 shows the structure factor in the absence of disorder, i.e., for $1/\tau = 0$, and we draw attention to the notable difference in the finite-τ cases.

The pair correlation function $g(r)$ is of equal physical interest. It is the probability that two particles may be found at a relative distance r, and can be obtained by Fourier transforming $S(q)$:

$$g(r) = 1 - 2r_s^{2/3} \int_0^\infty \mathrm{d}x \; xJ_0(x\tilde{r})[1 - S(x)] \tag{11}$$

where $J_0(x)$ is the zeroth-order Bessel function of the first kind, and $\tilde{r} = rq_s$. We note that $g(r)$ at small distances exhibits the familiar behaviour, namely that in the RPA it remains strongly negative for $r_s \gg 1$. This feature persists in the presence or absence of disorder. To remedy this deficiency one needs to go beyond the RPA; we discuss this in section 5.

3. A double-layer charged Bose system: effects of disorder

3.1. Plasmon dispersions

The density–density response for a two-component or two-layer system is defined by $\tilde{\rho}_i = \sum_j \chi_{ij} V_j^{\rm ext}$ where $\tilde{\rho}_i$ is the induced charge density for the ith species or the layer, and similarly V_j stands for the external perturbation. The RPA assumes that the induced charge density may be expressed as $\tilde{\rho}_i = \sum_j \chi^0_{ij}(V_j^{\rm ext} + V_j^{\rm eff})$, where the effective potential takes the form $V_j^{\rm eff} = \sum_k v_{jk}\tilde{\rho}_k$ in which v_{jk} are bare interactions. Combining the above, the inverse of the density–density response matrix is obtained as

$$[\chi(q,\omega)]^{-1} = \begin{pmatrix} [\chi^0_{11}(q,\omega)]^{-1} - v_{11}(q) & -v_{12}(q) \\ -v_{21}(q) & [\chi^0_{22}(q,\omega)]^{-1} - v_{22}(q) \end{pmatrix}. \quad (12)$$

The collective modes of the system are obtained by solving $\mathrm{Det}\,|\chi^{-1}| = 0$.

We first discuss the collective excitations of a single-layer, disordered, two-component charged Bose system in an analytically solvable model. In this case, $v_{11} = v_{12} = v_{22} = 2\pi e^2/q$ where we assume that particles interact via the long-range Coulomb potential. We also assume the same relaxation time τ for both components having the same mass m. They only differ in their number densities for which we define $\alpha = n_2/n_1$. These assumptions lead to the constraint $\chi^0_{22} = \alpha\chi^0_{11}$. For this simplified model the plasmon dispersions can be obtained analytically. We find

$$\omega_{\rm pl}/E_s = -\frac{\rm i}{2\tilde{\tau}} + \left[x^4 + \frac{x}{f_\pm} - \frac{1}{4\tilde{\tau}^2}\right]^{1/2} \quad (13)$$

where

$$f_\pm = \frac{1+\alpha}{2\alpha}\left[1 \pm \left[1 - \frac{4\alpha}{(1+\alpha)^2}\right]^{1/2}\right]. \quad (14)$$

Within the same model, analytical results can be obtained also for a double-layer system. The interlayer and intralayer Coulomb interactions are $v_{11} = 2\pi e^2/q$, and $v_{12} = [2\pi e^2/q]\exp(-qd)$, respectively, d being the separation distance between the two parallel layers. We do not allow any hopping or tunnelling between the layers. The resulting plasmon dispersions are given by exactly the same equation as above (equation (13)), with the replacement

$$f_\pm = \frac{1+\alpha}{2\alpha}\,\frac{1}{1-{\rm e}^{-2x\tilde{d}}}\left[1 \pm \left[1 - \frac{4\alpha}{(1+\alpha)^2}\,(1-{\rm e}^{-2x\tilde{d}})\right]^{1/2}\right] \quad (15)$$

where $\tilde{d} = dq_s$. Figure 5 shows the plasmon dispersions for the double-layer system in which the relaxation time τ is the same for both layers. Also shown by the dotted lines are the collision-free plasmon modes. Mode softenings for the two branches occur, as in the single-layer case. The above plasmon dispersion expressions further simplify if we consider two identical layers, i.e., $\alpha = 1$, to read [23]

$$\omega_{\rm pl}/E_s = -\frac{\rm i}{2\tilde{\tau}} + \left[x^4 + x(1 \pm {\rm e}^{-x\tilde{d}}) - \frac{1}{4\tilde{\tau}^2}\right]^{1/2}. \quad (16)$$

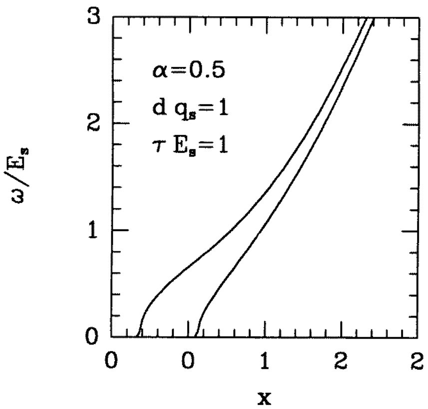

Figure 5. Collective modes of a double-layer, disordered CBG as a function of $x = q/q_s$. We have chosen the ratio of layer densities $\alpha = n_2/n_1 = 0.5$.

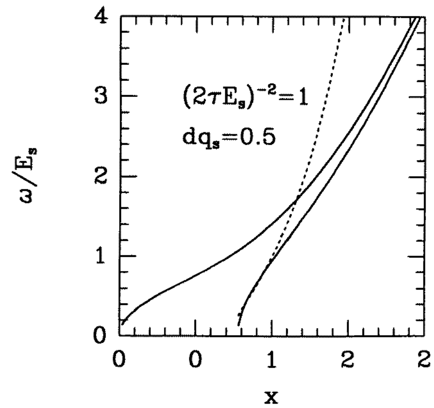

Figure 6. Collective modes of a double-layer CBG where one layer is collision-free and the other layer has $(2\tau E_s)^{-2} = 1$. The layer separation is $\tilde{d} = 0, 5$. Solid lines indicate the results of numerical calculation whereas the dotted line is from an analytic calculation of ω_B (see the text).

We now consider a scenario which may be regarded as a collisional analogue of the recently discussed [24] drag effect in a bilayer system. This is to consider a double-layer CBG system in which one of the layers is collision-free, and the other one includes the effects of collisions. We set out to study how the effects of collisions in one layer influence the other one. Assuming for simplicity that both layers are identical except for as regards the relaxation time τ considered for only one layer, the density response functions are given by

$$\chi_{11}^0 = \frac{2n\epsilon_q}{\omega^2 - \epsilon_q^2} \qquad \text{and} \qquad \chi_{22}^0 = \frac{2n\epsilon_q}{\omega\omega_+ - \epsilon_q^2}\,. \tag{17}$$

The collective modes of the system are obtained from the solution of a fourth-degree equation:

$$\tilde{\omega}^4 + \frac{\mathrm{i}}{\tilde{\tau}}\,\tilde{\omega}^3 - 2(x + x^4)\,\tilde{\omega}^2 - \frac{\mathrm{i}}{\tilde{\tau}}\,(x + x^4)\,\tilde{\omega} + x^8 + 2x^5 + x^2(1 - \mathrm{e}^{-2x\tilde{d}}) = 0 \tag{18}$$

in which $\tilde{\omega} = \omega/E_s$. We show the real parts of the plasmon dispersions in figure 6. There are two modes, one of which is undamped and starts from $x = 0$, while the other is damped and exists only above a critical wave vector. This is analogous to the drag effect in a bilayer electron system [24]. If we assume that a collective mode of equation (18) is of the form $\tilde{\omega} = \omega_B - \mathrm{i}\delta_B$, where the damping term δ_B is small, we obtain the following approximate expressions:

$$\begin{aligned} \omega_B^2 &\simeq x^8 + 2x^5 - 2x^2 p - (x + x^4)/4\tilde{\tau}^2 \\ \delta_B &\simeq -\frac{1}{\tilde{\tau}}(x + x^4 + \omega_B)/(x + x^4 - \omega_B) \end{aligned} \tag{19}$$

where $p = 1 - \mathrm{e}^{-2x\tilde{d}}$. The collective mode described by the above expressions is the damped plasmon obtained numerically above. We show ω_B in figure 6 by the dotted line, where it coincides with the numerical solution in the low-frequency regime, captured by the above analysis.

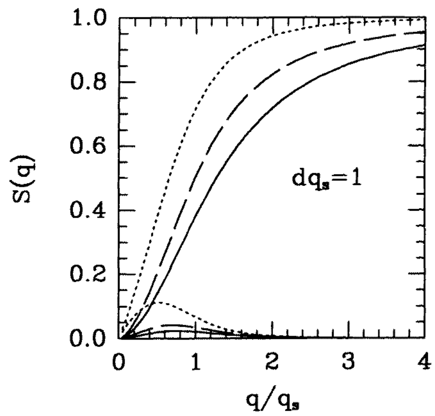

Figure 7. Static structure factors $S_{11}(q)$ and $S_{12}(q)$ for a double-layer CBG within the RPA and $(2\tilde{\tau})^{-2} = 0$, 1, and 5 shown by dotted, dashed, and solid lines, respectively.

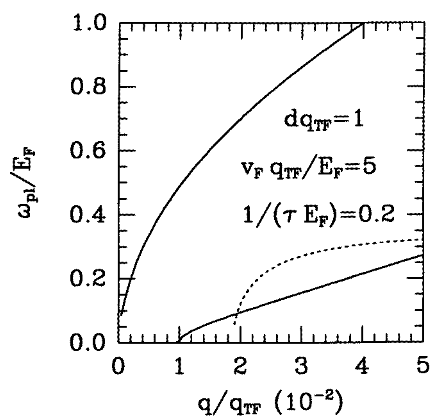

Figure 8. Collective modes of a double-layer electron gas where one layer is collision-free and the other layer has $1/(\tau E_F) = 0.2$ and $v_F q_{\mathrm{TF}}/E_F = 5$.

3.2. Static structure factors

The static structure factors for a double-layer system are evaluated in a manner similar to that for the single-layer system. Inverting the matrix (equation (12)) we obtain components of the density–density response function, and then we employ a relation of the form used by Zheng and MacDonald (see [21]). For the case of two identical layers with the same relaxation-time parameter τ, we obtain the following expressions for the intralayer and interlayer static structure factors, denoted by $S_{11}(q)$ and $S_{12}(q)$, respectively:

$$S_{11}(q) = \frac{1}{\pi}\frac{\epsilon_q}{\tau}\frac{1}{2}\left[I(\Delta_-) + I(\Delta_+)\right] \qquad S_{12}(q) = \frac{1}{\pi}\frac{\epsilon_q}{\tau}\frac{1}{2}\left[I(\Delta_-) - I(\Delta_+)\right] \tag{20}$$

in which $\Delta_\pm = 4[\epsilon_q^2 + 2n\epsilon_q(v_{11} \pm v_{12})] - 1/\tau^2$, and $I(\Delta)$ is defined in equation (10). As in the single-layer system, the expressions for S_{11} and S_{22} reduce to the disorder-free results [25] as $\tau \to \infty$. We display the effects of finite relaxation time τ on the intralayer and interlayer static structure factors for $\tilde{d} = 1$ in figure 7. The upper and lower curves are for $S_{11}(q)$ and $S_{12}(q)$, respectively. The dotted, dashed, and solid lines indicate $(2\tilde{\tau})^2 = 0$, 1, and 5, respectively. The intralayer and interlayer structure factors in the presence of disorder are lower than in the clean cases as would be expected.

4. Comparison with a disordered electron gas system

It will be of interest to compare the results for the model of a charged Bose bilayer, discussed in the previous section, with those for a corresponding electron gas bilayer. However, the dynamic polarizability of an electron gas layer is considerably more involved and hence we shall consider a long-wavelength and low-frequency approximation (i.e., $q \ll k_F$ and $\omega \ll E_F$, where k_F and E_F are Fermi wave vector and energy, respectively). We assume one of the layers to be clean and described by the polarization function

$$\chi_{11}^0(q,\omega) = -\frac{m}{\pi}\left[1 - \frac{\omega}{(\omega^2 - v_F^2 q^2)^{1/2}}\right] \tag{21}$$

and the other one is disordered with a polarization function given by [8–10]

$$\chi_{22}^{0}(q,\omega) = -\frac{m}{\pi}\left[1 - \frac{\omega}{(\omega_{+}^{2} - v_{F}^{2}q^{2})^{1/2} - \mathrm{i}/\tau}\right] \tag{22}$$

where v_F is the Fermi velocity assumed to be the same for both layers. Calculating the collective modes numerically by solving the mode equation [26]

$$1 - v_q(\chi_{11}^{0} + \chi_{22}^{0}) + p v_q^{2}\chi_{11}^{0}\chi_{22}^{0} = 0 \tag{23}$$

where $p = 1 - \exp(-2qd)$, we obtain two solutions. Figure 8 displays the numerically obtained collective modes in a double-layer electron gas. We observe that one mode starts from the origin (i.e., is unaffected by disorder) and behaves as $\sim q^{1/2}$ (the optical mode), whereas the other mode exists only above a critical wave vector and exhibits a linear q-dependence (the acoustic mode).

It may be recalled that for the special case of a disordered two-layer electron system with the same relaxation time τ, and Fermi velocity v_F, but different masses, a closed-form solution for the collective modes is possible in the small-q limit [9]. It was found that the two plasmon modes (optical and acoustic) can only propagate above certain critical wave vectors. In other words, both the modes are affected by disorder.

5. Discussion

We have assumed without proof that there is a condensate in a 2D-CBG both in the absence and in the presence of disorder. There has been some recent theoretical work [27] on the question of CBG condensate in two-dimensional systems. Pitaevskii and Stringari [27] derive an inequality for the momentum distribution function n_q which involves the structure factor $S(q)$. Magro and Ceperley [28] use a combination of the findings in [27] and a diffusion Monte Carlo method to argue that the condensate fraction n_0 will be zero for a 2D-CBG with a $(\ln r)$-potential. However, it seems that their argument could formally be applied also to a 3D-CBG which does have a condensate. Furthermore, the derivation in [27] assumes that $\omega(q)$ has a gap. It is known that in 2D the plasmon dispersion $\omega(q)$ is gapless for a $(1/r)$-potential, but not for a $(\ln r)$-potential. In the presence of disorder $\omega(q)$, still gapless, develops a cut-off wave-vector q_c, i.e., $\mathrm{Re}[\omega(q)]$ is zero below q_c. Additionally, for a $(\ln r)$-potential in contrast to a $(1/r)$-potential, the plasmon dispersion has a gap as in a 3D-CBG. In view of these considerations we feel that the question of a condensate in a 2D-CBG is still an open problem. For a two-layer system, the interlayer Coulomb coupling brings in another element that would make it different from a strictly 2D system. In fact, it can be regarded as an intermediate between a bulk 3D system and a 2D single-layer system. Therefore, the arguments developed in the context of a condensate for a single-layer 2D system need not strictly apply to the case of a two-layer system. With these provisos we have assumed a condensate along the lines given by Alexandrov and Mott (see [2]), and of Gold [4, 23].

In the foregoing analysis of the plasmon dispersions we have chiefly used the RPA. An intuitively appealing way of including the exchange–correlation effects is through the local-field factor which has been extensively used for the electron gas. They are also important in the CBG; for instance, a roton-like structure appears [24] in the plasmon dispersion at low density ($r_s \gg 1$) when a local-field factor is introduced. The local-field concept is on the same level of semi-phenomenological approximation as the relaxation-time scheme discussed in the previous sections [9]. A simple approximation assuming a delta-function form (in real space) for the exchange interaction [29] leads to a modification of the

RPA dielectric function with the replacement of the bare interaction $v(q)$ by $v(q) - J/2$. Here J is a constant (in q-space) and may be identified with the local-field factor through $v(q)G(q) = J/2$. It is interesting to note that the resulting plasmon dispersion for a single-layer, disordered CBG using this local-field factor is

$$\omega_{\mathrm{pl}}/E_s = -\frac{\mathrm{i}}{2\tilde{\tau}} + \left[x^4 + x - \tilde{J}x^2 - \frac{1}{4\tilde{\tau}^2}\right]^{1/2} \tag{24}$$

where $\tilde{J} = nJ/E_s$. When we employ the logarithmic potential (cf. section 2), the above dispersion relation is altered only in the linear term where it is replaced by unity. The critical wave vector, x_c, below which the plasma excitations are not supported for the logarithmic potential is now given by

$$x_c = \left[\frac{\tilde{J}}{2} + \left[\frac{\tilde{J}^2}{4} - \left(1 - \frac{1}{4\tilde{\tau}^2}\right)\right]^{1/2}\right]^{1/2}. \tag{25}$$

On the other hand, the Hubbard approximation to the local-field factor appropriate for a clean 2D-CBG is given by [23] $G_H = r_s^{2/3}x/(1+x^2)^{1/2}$. We neglect the effects of scattering from disorder on $G(x)$, as in earlier calculations [9, 23]. With this approximation, the plasmon dispersion for a single-layer, disordered, CBG interacting via a Coulomb potential becomes

$$\omega_{\mathrm{pl}}/E_s = -\frac{\mathrm{i}}{2\tilde{\tau}} + \left[x^4 + x\left[1 - \frac{r_s^{2/3}x}{(1+x^2)^{1/2}}\right] - \frac{1}{4\tilde{\tau}^2}\right]^{1/2}. \tag{26}$$

We note that the long-wavelength behaviour of ω_{pl} is little affected, whilst the local-field effects will soften the plasmon mode at larger values of q. The critical wave vector x_c, below which plasmon excitations are not supported, is obtained from the modified equation

$$x_c^4 + x_c\left[1 - \frac{r_s^{2/3}x_c}{(1+x_c^2)^{1/2}}\right] - \frac{1}{4\tilde{\tau}^2} = 0. \tag{27}$$

Inclusion of local-field effects increases x_c in general, for a given $\tilde{\tau}$, compared with the result without the local-field factor. Similar trends are expected in the case of a double-layer disordered CBG.

We note that the effects of the local-field corrections may also be incorporated in our analytic expressions for the static structure factors, and more sophisticated theories may be constructed. Local-field factors and $S_{ij}(q)$ have been calculated self-consistently for a double-layer CBG in the absence of disorder [25]. Our analytical results for plasmon dispersion relations and static structure factors for the single- and double-layer disordered Bose systems may be used as input in more improved many-body theories such as ground-state calculations, disorder-induced superfluid–insulator transitions, and charge-density-wave instability in Bose systems. Finally, our results may also be helpful in distinguishing the Fermi and Bose liquid models of high-T_c superconductors.

In summary, we have studied some new features of the collective excitations in a single- and double-layer charged Bose gas in the presence of disorder whose effect has been taken into account through collisions in the relaxation-time approximation. We have found that there is a critical wave vector below which plasmons do not propagate, as in the electron gas case. More varied results are obtained if in a two-layer system one of the layers is disordered while the other layer is disorder-free. We have investigated the role of the boson interaction potential for the collective excitations. Closed-form expressions are given for the static structure factor $S(q)$ in single- and double-layer systems which may be useful for subsequent applications. Many-body corrections beyond the RPA are also briefly mentioned.

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